

Three dimensional aerodynamic characteristics
of weis-fogh mechanism[†]

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(Manuscript Received July 27, 2007; Revised January 19, 2008; Accepted July 31, 2008)

Abstract

The three-dimensional flows in the Weis-Fogh mechanism are studied by flow visualization and numerical simulation by the discrete vortex method. The vortex method, especially the vortex stick method, is employed to investigate the vortex structure in the wake of the two wings. The pressure is estimated by the Bernoulli equation, and the lift on the wing is also obtained. For the results, the eddies near the leading edge of each wing in the fling stage take a convex shape because the eddies shed from both tips entrain the flows, and the downwash in the rotating stage is deflected toward the outside because the outside tip vortex is stronger than the inside one. The lift coefficient on the wings in this mechanism is almost independent of the Reynolds number.

Keywords: Computational fluid dynamics; Vortex method; Unsteady flow; Vortex method; Random walk method

1. Introduction

The Weis-Fogh mechanism, discovered by analyzing the wing motion in the hovering flight of a small insect called *Encarsia formosa*, is an efficient mechanism of lift generation [1]. The motion of the wings in the Weis-Fogh mechanism as shown in Fig. 1 may consist of five stages. The first is the “clap” stage, in which the two wings close touching their leading edges. The second is the “fling” stage, in which the two wings open touching their trailing edges. The third is the “rotating” stage, in which the two wings rotate in the horizontal plane with the center corresponding to the insect’s body. The fourth is the “flap” stage, in which the wings change the angle of attack, and the last is the “returning” stage, in which the wings return to the position of the “clap” stage. These stages are repeated and the lift on each wing is always

generated in the upward direction. In this figure, the insect is on the hovering flight and the arrows indicate the direction of circulation around wings.

Lighthill [2] studied the two-dimensional flow in this mechanism by the potential theory. Maxworthy [3], through the two-dimensional modeling experiment of this mechanism, visualized the circulation which happened around the wing at the fling stage, also sketching the vortex around the wing as for the three-dimensional flow. However, there had not been

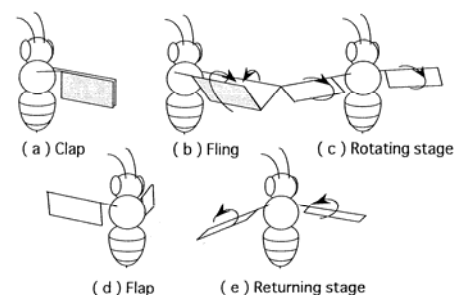


Fig. 1. Motion of the wing in the Weis-Fogh mechanism.

[†] This paper was recommended for publication in revised form by Associate Editor Haecheon Choi

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any theoretical study on the three-dimensional flow of Weis-Fogh mechanism, also the visualization experiment being ended up only at the stage of fling. Hence, in this study, it was aimed to clearly understand the three-dimensional flow around the wing at the fling and the following rotating stages, by way of the visualization experiment and the numerical calculation utilizing the discrete vortex method. The flow around the wing, as anticipated in Fig. 1, was fully three-dimensional and unsteady. As wing movement is also complicated, the finite element method as well as the finite-difference method, which must construct meshes in every time step, are not suitable for this kind of simulation. Therefore, in this numerical calculation, the discrete vortex method [4, 5], which did not need the application of a mesh and was useful for separation flow, was decided to be introduced. Also, in this calculation, the viscous diffusion of the fluid was expressed by Random walk method [6].

2. Flow visualization

2.1 Experimental method

The flow patterns around the wings were visualized by titanium tetrachloride, which generates a thick mist of HCl. The apparatus for the experiments was as follows. The wings were made of acryl and each had a square shape with a 70-mm chord length, a 140-mm span, and a 2-mm thickness. The two wings were set to move symmetrically, and the motion was controlled by hand. The rods to control the wings' motion were connected to wings at their trailing edges as shown in Fig. 2, unlike Maxworthy's experiments [3], in which they were connected with the wings at their leading edges. Therefore, the wings were rotated about their trailing edges in the fling stage, and they translated (or rotated) with their trailing edges leading. The edges and the tips of the wings were coated with

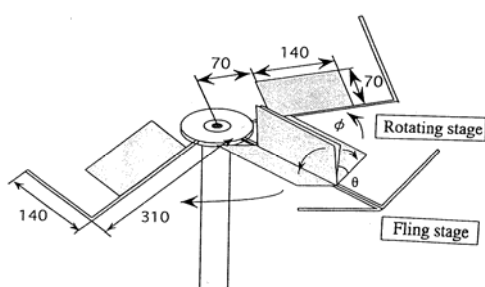


Fig. 2. Schematic diagram of experimental apparatus (unit : mm).

TiCl_4 just before the wings opened. The flow patterns were recorded by a video camera.

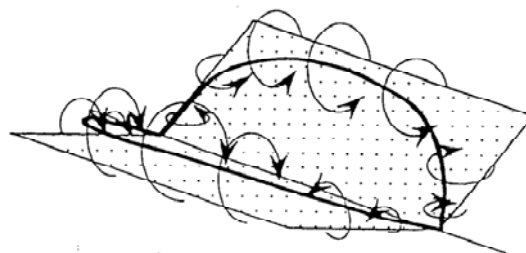
2.2 Experimental results and discussion

The flow patterns in the fling stage only are shown in Fig. 3, in which (a) is the photograph of a flow visualization and (b) is a sketch of the structure of vortices under the same condition as is (a). The Reynolds number is defined by $\text{Re} = \Omega c^2 / \nu$, where Ω is the angular velocity of rotation in the fling, c is the chord length, and ν is the kinematic viscosity. In this figure, the eddies formed by the shearing layers shed from the leading edges are located near the leading edges. These eddies are known to increase the lift in the fling stage. The eddies near the leading edge of each wing take a convex shape because the eddies shed from both tips of the wings entrain the flows.

The flow pattern in the rotating stage, as shown in Fig. 4, becomes complicated due to the interaction among the eddies from the leading edges and the tips. The strongest eddies are those from the leading edges, and the eddy from one wing is connected to the one from the other wing. The flow is formed by three-dimensional vortices, and a sketch is presented in Fig. 4(b)

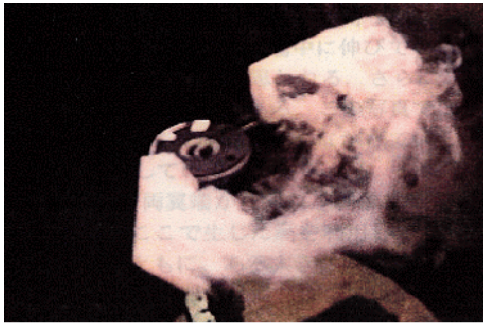


(a) Photograph

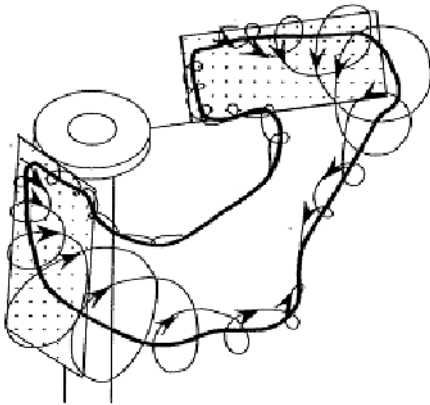


(b) Sketch of vortex structure

Fig. 3. Visualization of flowfield around the wings in the fling stage ($\text{Re} = 400$, $\theta = \pi/3$).



(a) Photograph



(b) Sketch of vortex structure

Fig. 4. Visualization of flowfield around the wings in the rotating stage ($Re = 400, \theta = \pi/3, \phi = \pi/3$).

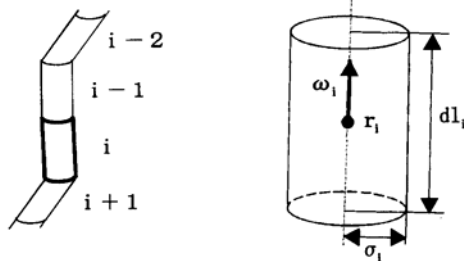


Fig. 5. Vortex filament approximated by vortex stick.

3. Numerical study

3.1 Method of simulation

In this study, the so-called vortex stick method was employed, in which all the vortex filaments are divided into several cylindrical vortex sticks as expressed in Fig. 5 [7]. In the figure, the i -th vortex stick was defined with the 8 quantities such as $\mathbf{r}_i(x, y, z)$, the position of the center, $\boldsymbol{\omega}_i(\omega_x, \omega_y, \omega_z)$, the vector

of vorticity, σ_i , the radius of vortex core and δl_i , the length, while the change of each element with respect to the time was obtained as in the following order.

Firstly, \mathbf{u}_{ij} , the induced velocity at the central position of the i -th vortex stick, was obtained by Biot-Savart's Law as follows.

$$\mathbf{u}_{ij} = \frac{1}{4\pi} \int \frac{\boldsymbol{\omega}_j \times (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^3} d\mathbf{r} \quad (1)$$

$\mathbf{r}_i^{t+\Delta t}$, the central position of the i -th vortex stick, was moved in every time step Δt by

$$\mathbf{r}_i^{t+\Delta t} = \mathbf{r}_i^t + \mathbf{u}_i^t \Delta t = \mathbf{r}_i^t + \sum_{j=1}^n \mathbf{u}_{ij}^t \Delta t \quad (2)$$

Also, the time variation of the vorticity $\boldsymbol{\omega}_i$, was calculated from the vorticity transport equation, which omitted the viscous term as follows.

$$\frac{D\boldsymbol{\omega}_i^{t+\Delta t}}{Dt} = \boldsymbol{\omega}_i^t \cdot \nabla \mathbf{u}_i^t = \boldsymbol{\omega}_i^t \nabla \left(\sum_{j=1}^n \mathbf{u}_{ij}^t \right) \quad (3)$$

In the meanwhile, the time variations of σ_i , the radius of vortex core, and δl_i , the length were obtained from the theorem of Kelvin and from the continuity equation, respectively, as follows.

$$|\boldsymbol{\omega}_i^{t+\Delta t}| \pi (\sigma_i^{t+\Delta t})^2 = |\boldsymbol{\omega}_i^t| \pi (\sigma_i^t)^2 \quad (4)$$

$$\pi (\sigma_i^{t+\Delta t})^2 \delta l_i^{t+\Delta t} = \pi (\sigma_i^t)^2 \delta l_i^t \quad (5)$$

From the above Eqs. (2), (3), (4) and (5), the time variations of 8 quantities of vortex stick were able to be obtained, while the viscous diffusion term was approximated by the random walk method. Namely, the displacement vector was obtained from the regular random number with an average value of 0 (zero) and a dispersion of $2\nu\Delta t$ (ν : kinematic viscosity), being added to the right side of Eq. (2).

The motions of the two wings are completely symmetrical, so that only one wing was considered, and the entire flowfield was obtained by the assumption of symmetry as shown in Fig. 6. Each wing was represented by 10×20 square panels. Each panel has a ring-type line vortex, represented by the wavy lines in Fig. 7, and then is finally replaced with vortex

sticks as shown in Fig. 7. The circulation of each vortex stick is evaluated by the difference between the circulations of two ring-type vortices that the vortex stick contains. The middle in span direction and a quarter in chord direction of each panel was taken as the control point at which the boundary condition was applied. The boundary condition for determining the circulations of the ring-type vortices was that the normal components of the velocities of the fluid and those of the wing at the control points coincide with each other.

The vortex sticks at the edges and tips of the wing are shed into the flow in each time step, maintaining their circulations. The structure of the vortices in the flowfield is also ring-like as shown. All the vortex lines are connected and no vortices disappear.

If the vorticity of the ring-type line vortices on the surface of the wing is assumed to be concentrated on the cores, then the flow on the surface, excluding the cores, can be considered as potential flow. Therefore, the pressure distribution on the wing can be evaluated by the Bernoulli equation for unsteady flows

$$\frac{\partial \phi}{\partial t} + \frac{u^2}{2} + \frac{p}{\rho} = const \tag{6}$$

where ϕ is the velocity potential, u is the local

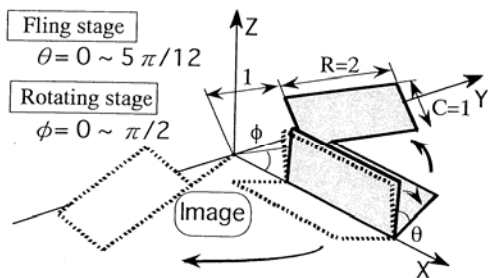


Fig. 6. Three-dimensional analytical model.

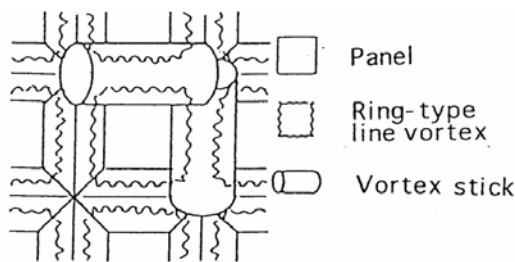


Fig. 7. An enlarged diagram of panel.

velocity, p is the pressure, and ρ is the density of the fluid. For calculating the forces on the wing, only the forces due to the pressure are considered.

The calculation method for the force on the wing is shown in Fig. 8. The wing is approximated as a flat plate without thickness, so that only the difference between the pressures on both sides of the wing should be considered. Only the first term in Eq. (6) is different, that is, this term is multi-valued, but the other terms, which are single-valued, are not different between the two sides. Therefore, the first term will be evaluated here. For example, since the difference of the velocity potential between both sides of the 'a' point in Fig. 8, which is $(\Phi_{a1} - \Phi_{a2})$, corresponds to the circulation Γ_a of the ring-type line vortex, the

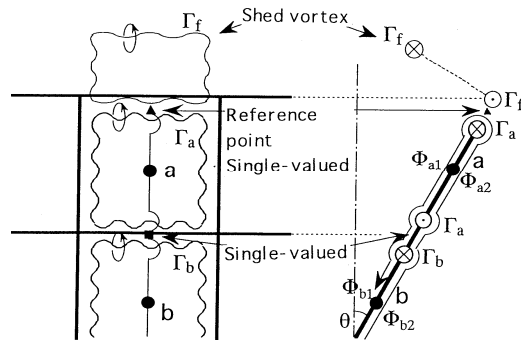


Fig. 8. The calculating method for force on the wing.

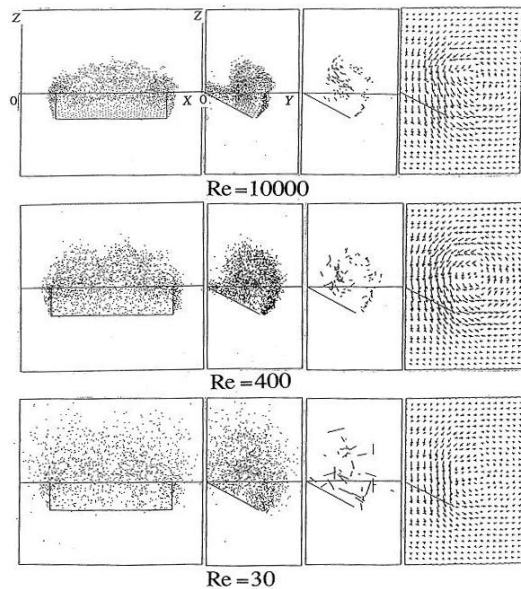


Fig. 9. Flowfield around the wing in the fling stage ($\theta=2\pi/3$).

first term of Eq. (6) on that point can be obtained by estimating the variation of the circulation with time.

$$\frac{\Phi_{a1}}{\partial t} - \frac{\Phi_{a2}}{\partial t} = \frac{\Gamma_a^t - \Gamma_a^{t-\Delta t}}{\Delta t} = \dot{\Gamma}_a \quad (7)$$

As the same procedure, that on the 'b' point is expressed as

$$\frac{\Phi_{b1}}{\partial t} - \frac{\Phi_{b2}}{\partial t} = \frac{\Gamma_b^t - \Gamma_b^{t-\Delta t}}{\Delta t} = \dot{\Gamma}_b \quad (8)$$

The force on the perpendicular direction to the surface of the wing F is calculated by

$$F = \int pds = -\rho \sum_{j=1}^n \dot{\Gamma}_j \Delta s_j \quad (9)$$

where the term $\dot{\Gamma}_j$ represents the time-variations of the circulations in the j-th panel, which is evaluated as shown in Eq. (7) and Δs_j is the area of the panel. All the Γ_j on the surface affect the force and they are ring-type. Therefore, the force remains the same whether it is calculated the spanwise or chordwise. The lift, which is the z-direction component of the force in Fig. 6, is expressed as

$$L = -\rho \left(\sum_{j=1}^n \dot{\Gamma}_j \Delta s_j \right) \sin \theta \quad (10)$$

where θ is the opening angle shown in Fig. 6.

3.2 Numerical results and discussion

For easy comparison with the experimental results, the calculating conditions were made similar to the experimental conditions. The chord length c and the span R of the wing are 1.0 and 2.0, respectively; the time step Δt is 0.05; the angular velocity Ω is 1.0 both in the fling and rotating stages; and the distance from the z-axis to the middle of the wing span is 2.0. The results are represented as a function of the Reynolds number Re , the opening angle θ , and the rotating angle ϕ .

The flow patterns for various Reynolds numbers in the fling stage are shown in Fig. 9. The figures in the first and second columns represent the distributions of the vortex sticks in the z-x and z-y planes, respectively; and the third and last columns show the stick traces and the velocity fields in the z-y plane in the

cross section at the middle of the wing span, respectively.

The distributions of the vortex sticks and the stick traces are scattered more at low Reynolds numbers. This explains the effect of the viscous diffusion. In the first column, the distribution of the vortex sticks is convex in structure, and this corresponds to the results obtained by flow visualization. The velocity fields show that a large circulation in the opposite direction to the opening is formed behind the leading edge of the wing.

The flow patterns for various opening angles in the fling stage are shown in Fig. 10. A large structure of the vortex shed from the leading edge is located near the leading edge as seen from the distribution of vortex sticks, which agrees well with the results of flow visualization shown in Fig. 3. The stick traces show that flow separates from the leading edge and reattaches to the wing with time.

The lift coefficient $C_L = L / (\frac{1}{2} \rho \Omega^2 c^2 S)$ in the fling stage is shown in Fig. 11 as a function of the opening angle. The large fluctuations of the lift coefficients in the Fig. 11 are because the unsteady force

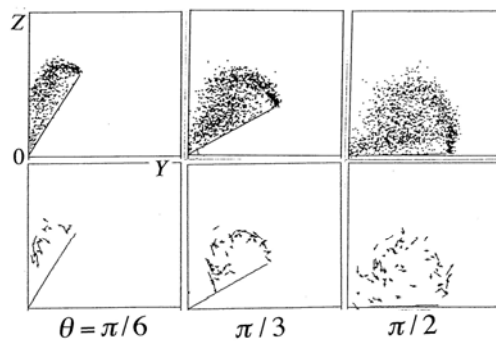


Fig. 10. Vortex sticks and stick traces around the wing in the fling stage ($Re = 400$).

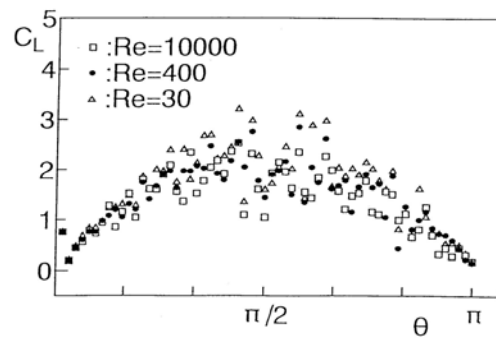


Fig. 11. Lift coefficient in the fling stage.

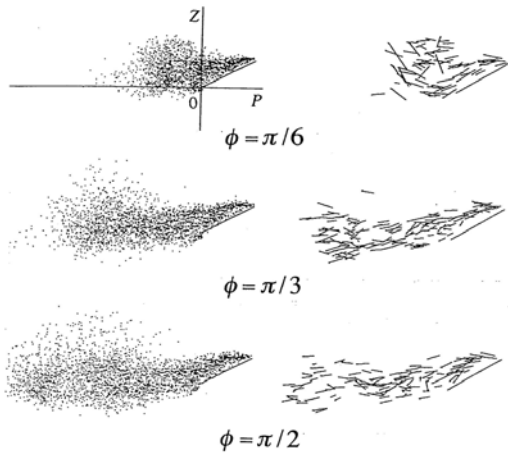


Fig. 12. Vortex sticks and stick traces around the wing in the rotating stage ($Re = 400$, $\theta = \pi/3$).

of the flow is acting very sensitively on the wing, for the wing is approximated as a flat plate without thickness in the calculation.

The maximum of the lift, regardless of the Reynolds number, occurs at an opening angle of about $\pi/2$. It is difficult to compare exactly these results with the experimental results by Spedding and Maxworthy [8] because the angular velocity in their experiments changes with time. But the calculation results agree qualitatively with the experimental results because the lift is maximum at an opening angle of $\theta = \pi/2$ and minimum at $\theta = 0$ and $\theta = \pi$.

The flow patterns for various rotating angles are shown in Fig. 12. In this calculation, the wing opens up to $\pi/3$ and moves to the rotating stage. In this figure P, which is fixed on the wing as shown in Fig. 12, represents an axis of coordinates in the horizontal and perpendicular plane to the span direction of the wing.

As the case was for the rotating stage, the separation happened at the leading and trailing edges of the wing, while the distribution of vortex stick became longer as the wing rotated.

Under the same conditions as in Fig. 12, the flow patterns in the Z-Q plane are shown in Fig. 13. In this figure, Q, which also is fixed on the trailing edge of the wing in the rotating stage, represents the span direction of the wing. The velocity fields in the second column show that the flow near the center of the upper surface of the wing is accelerated by the tip vortices in the opposite direction. The velocity fields show that the downwash is deflected toward the outside because the outside tip vortex is stronger than the

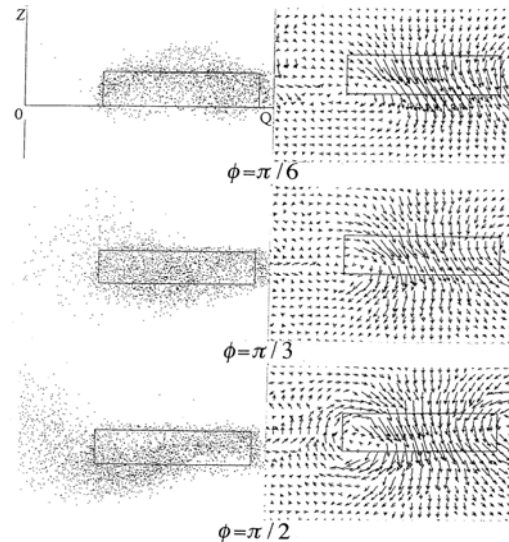


Fig. 13. Vortex sticks and velocity vectors around the wing in the rotating stage ($Re = 400$, $\theta = \pi/3$).

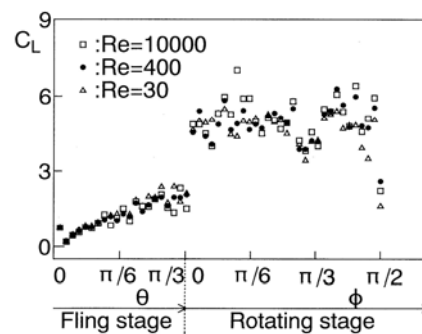


Fig. 14. Lift coefficient in the fling and rotating stages ($\theta = \pi/3$).

inside one. In the first column, the vortices of the convex type at the rotating angle $\phi = \pi/6$ flow along the trailing edge with an increase of the rotating angle ϕ because of the downwash by tip vortices. In the first columns of Fig. 12 and 13, the vortex sticks are distributed in the direction opposite to the moving direction of the wing. This means that the vortices between the wings are connected to each other, which agrees well with the results of the flow visualization shown in Fig. 4.

The lift coefficients for various Reynolds numbers in the fling and rotating stages are shown in Fig. 14. This figure shows that the lift coefficient is almost independent of the Reynolds number in the rotating stage as well as in the fling stage. In this study, the angular velocity in the rotating stage Ω is unity as well as that in the fling stage. Therefore, the moving

velocity of the edge at the middle of the span in the rotating stage is twice as large as that in the fling stage. If the lift coefficient is nondimensionalized by the middle speed of the wing span in the rotating stage, this coefficient in the rotating stage should be a quarter of the present value.

4. Conclusions

The three-dimensional flows in the Weis-Fogh mechanism were studied by flow visualization and numerical simulation by using the vortex method. In this study, the viscosity of the fluid was represented by the random walk method.

The flow patterns which are widely different from those in the two-dimensional flow, are strongly influenced by the tip vortices.

The eddies near the leading edge of each wing in the fling stage take a convex shape because the eddies shed from both tips entrain the flows.

The downwash in the rotating stage is deflected toward the outside because the outside tip vortex is stronger than the inside one.

The lift coefficient on the wings in this mechanism is almost independent of the Reynolds number.

The discrete vortex method is very effective for simulating this type of flow.

Acknowledgments

This work was supported by the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government (MOST) (No. R01-2007-000-10038-0).

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